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ESTIMATION OF PERTURBATIONS DUE TO FLOW AROUND BLAST GAUGES WITH SPHEROIDAL SHAPES

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1. Introduction. When a pressure gauge is placed in the field of a blast wave in a medium such as air or water, the pressure record is affected(1 by the deflection around the gauge of the moving particles in the medium. In other words, the gauge perturbs the pressure field that it is supposed to measure. In a previous memorandum (henceforth referred to as I)(2 estimates were given for the perturbations due to the deflection, by various shapes of baffles, of the approximately steady flow following a shock wave. It was shown that the perturbation in the pressure record may be reduced by placing a suitable baffle around the gauge, as indicated in Fig. 1.

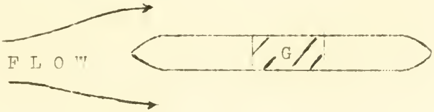

Fig. 1 Pressure gauge G with baffle

(1 Arons, Tait, Fraenkel and Deane "Characteristics of Air Blast Gauges" NDRC Report AES-8a(OSRD 4875a) March 1945

(2 MacDonald and Schaaf "On the Estimation of Perturbations Due to Flow Around Blast Gauges" AMP Note 22 (AMG-NYU 136) September 1945.

In making pressure-time oscillograms for non-plane blast waves (such as are produced by bombs in the open) it is the usual practice, because of buffeting effects, to use tourmaline pressure gauges without baffles. Under such circumstances the pressure perturbation varies with position on the gauge which then responds to an average of the pressure over its surface. It is the purpose of this memorandum to estimate the pressure perturbation at various points on a gauge considered as an oblate spheroid of suitable size, placed edge on to a steady compressible subsonic flow. [From conversations with Dr. Arons of Division 2 NDRC (Woods Hole) it appears that although the pressure behind the shock front in air blasts from bombs is not steady like that in shock tubes, nevertheless the assumption of steady flow conditions immediately behind the shock front leads to calculated values of peak pressures in agreement with those predicted by pressure-distance formulas. Therefore transient effects are apparently unimportant in the determination of peak pressures.]

2. Results.(3 (see Appendix for the formulas used).

(a) The percentage pressure decrease $\delta p = -100\Delta p/p_o$ at any point P on the surface of the gauge depends on the Mach number(4 of the incident flow and on a shape factor, $C_{p,o}(P)$, called the pressure coefficient. For subsonic flow the following is a good approximation:

$$(1)^{(5} \qquad \delta p = \frac{-70\, M^2}{\sqrt{1-M^2}}\, C_{p,o}(P) \quad (70 \text{ is } \tfrac{1}{2} \cdot 100 \cdot \gamma)$$

(3 The notation throughout is the same as in I.

(4 The ratio of the velocity of the undisturbed flow to the velocity of sound. For a shock wave moving into resting air at absolute pressure p_o we have $M^2=(1-p_o/p)^2/(.28+1.68p_o/p)$ where p is the absolute pressure behind the shock.

(5 Derived in I, page 9.

The value of this pressure coefficient $C_{p,o}(P)$ at indicative points on the surface of two oblate spheroids of thickness $t = 3/13$ and $t = 3/7$ respectively[6] is given schematically in Figs. 2 and 3 on the next two pages. The points on the surface are projected on the equatorial plane of the spheroid. The distribution of $C_{p,o}(P)$ is given over only one quadrant, since it is symmetric with respect to both axes. The incident flow is assumed to be parallel to the equatorial plane of the spheroid corresponding to the "edge on" orientation of the gauges. To find the percentage decrease in pressure at any point on the gauge surface, for a given velocity of incident flow, equation (1) may be used.

(b) The response given by a tourmaline gauge when the pressure varies over the surface is a weighted average of some sort taken over the entire surface of the tourmaline crystal. We hence seek an appropriate mean pressure coefficient $\bar{C}_{p,o}$, corresponding to this average surface.

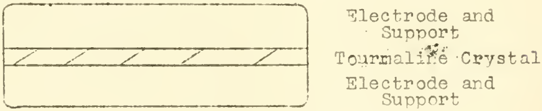

Fig. 4 Cross-section of a tourmaline gauge

Since the top and bottom faces of the crystal - which are cut perpendicular to the optic axis - are about four times as sensitive to pressure as the side, and also have a very much greater area, we ignore the side effects entirely.

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[6] By an oblate spheroid of thickness t we mean an ellipsoid whose semi-axes are a:a:at.

The proper average to use over the top and bottom faces might at first thought be taken to be the simple unweighted average of $C_{p,o}(P)$ over the equatorial plane circle of the corresponding spheroid. However (see Fig. 5), the spheroid which

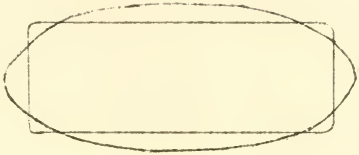

Fig. 5 Gauge and approximating spheroid

most nearly approximates the gauge extends beyond the top and bottom faces of the tourmaline crystal. This suggests that we take the average of $C_{p,o}(P)$ over only a part of the area of the corresponding equatorial circle. Inspection of of Fig. 2 and 3 shows that $C_{p,o}(P)$ is quite constant over a large portion of the equatorial circle, changing appreciably only near the leading and trailing edges - i.e. in just the regions to be omitted in calculating the average $\bar{C}_{p,o}$. Accordingly, we expect $\bar{C}_{p,o}$ to have values something like -.35 and -.60 for t = 3/13 and 3/7 respectively. These are the values used in calculating Table I.

(c) In the case of a true spheroidal gauge, we would

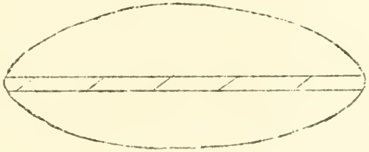

Fig. 6 Spheroidal Gauge

expect $\bar{C}_{p,o}$ to be the average of $C_{p,o}(P)$ over the entire equatorial circle. These values have been calculated (see appendix) to be -.27 and -.43 for $t = 3/13$ and $3/7$ respectively.

Table I: For a blast wave the idealization corresponds to the following:

M = Mach number of flow behind shock front

$\bar{\delta}p$ = average percent perturbation of peak pressure recorded by gauge

t = ratio of axes of spheroid

M	$\bar{\delta}p(t=3/13)$	$\bar{\delta}p(t=3/7)$	$70M^2/\sqrt{1-M^2}$
.1	.25	.42	.7
.2	1.0	1.8	2.9
.3	2.3	4.0	6.6
.4	4.2	7.5	12.2
.5	7.0	12	20
.6	11	19	31
.7	17	29	48

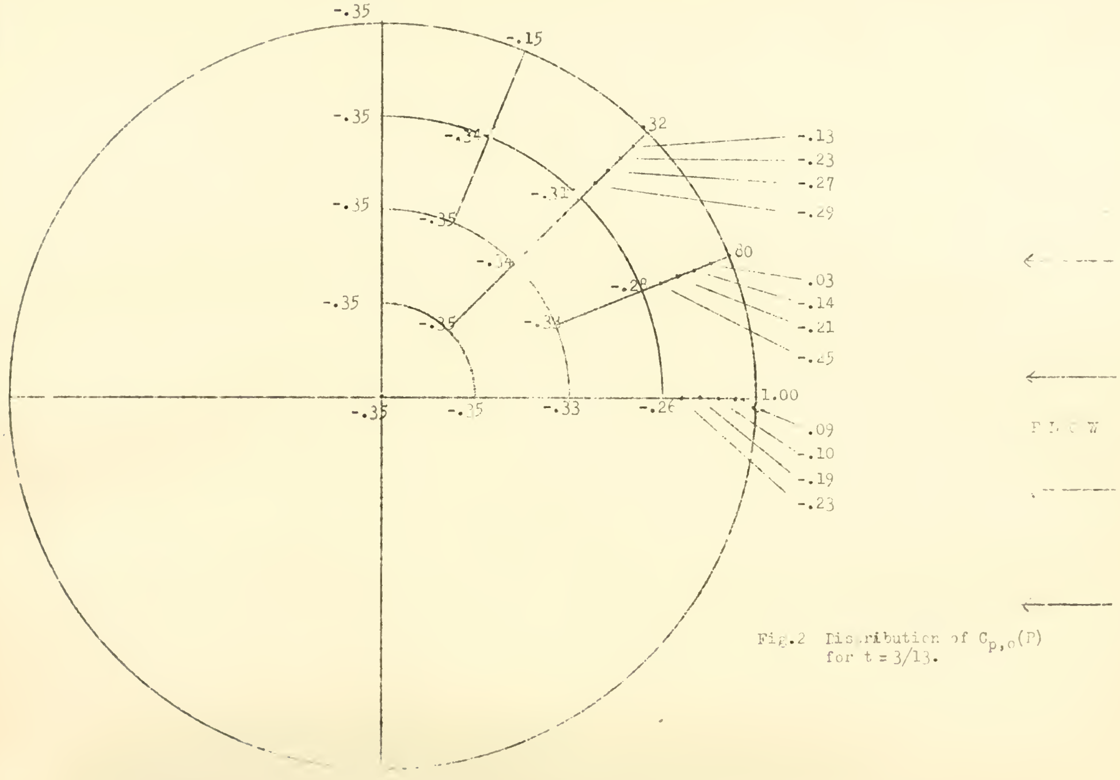

Fig.2 Distribution of $C_{p,o}(P)$
for t = 3/13.

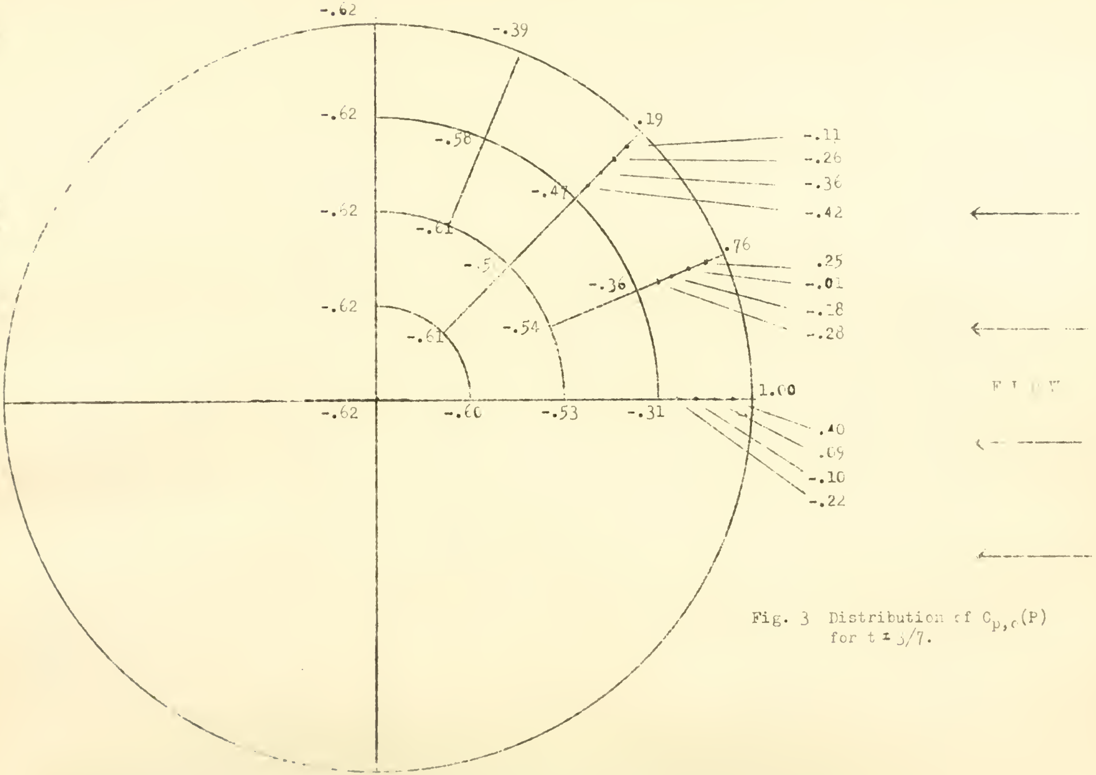

Fig. 3 Distribution of $C_{p,c}(P)$ for t = 3/7.

3. Appendix

(a) Calculation of $C_{p,o}(P)$.

The velocity potential $\Phi(x,y,z)$ of the incompressible flow around an ellipsoid

$$(2) \qquad \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{a^2t^2} = 1$$

is(7

$$(3) \qquad \Phi = ux\left\{1 + \frac{a^3t}{2-\alpha_o}\int_\lambda^\infty \frac{d\sigma}{(a^2+\sigma)^2(a^2t^2+\sigma)^{1/2}}\right\}$$

where λ is defined by

$$(4) \qquad \frac{x^2+y^2}{a^2+\lambda} + \frac{z^2}{a^2t^2+\lambda} = 1\ ,$$

and

$$(5) \qquad \alpha_o = a^3t\int_o^\infty \frac{d\sigma}{(a^2+\sigma)^2(a^2t^2+\sigma)^{1/2}}$$

$$= \frac{t}{1-t^2}\left\{\frac{\cos^{-1}t}{\sqrt{1-t^2}} - t\right\}$$

and u is the velocity of the undisturbed flow, which has been taken to be in the direction of the x-axis. The speed q at any point is given by

$$(6) \qquad q^2 = \Phi_x^2 + \Phi_y^2 + \Phi_z^2$$

(7 Milne-Thomson, "Theoretical Hydrodynamics" page 456.